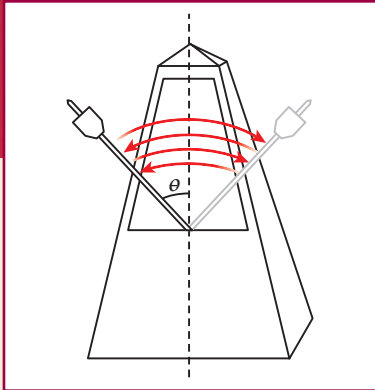


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# Vibrations and Waves



A mechanical metronome consists of an inverted pendulum and a counterweight on opposite sides of a pivot. A sliding weight above the pivot is used to change the rate of vibration. As the pendulum vibrates, the metronome ticks, and musicians use the sound to keep a steady tempo. This vibration is an example of a *periodic motion*.

## WHAT TO EXPECT

In this chapter, you will study a kind of periodic motion called *simple harmonic motion* and will learn about the relationship between simple harmonic vibrations and waves.

## WHY IT MATTERS

Waves can carry information, such as conversations and television broadcasts. Much of your perception of the physical world is dependent on waves. You could not hear or see anything without sound waves and light waves.

## CHAPTER PREVIEW

### 1 Simple Harmonic Motion

Hooke's Law  
The Simple Pendulum

### 2 Measuring Simple Harmonic Motion

Amplitude, Period, and Frequency

### 3 Properties of Waves

Wave Motion  
Wave Types  
Period, Frequency, and Wave Speed

### 4 Wave Interactions

Wave Interference  
Reflection  
Standing Waves



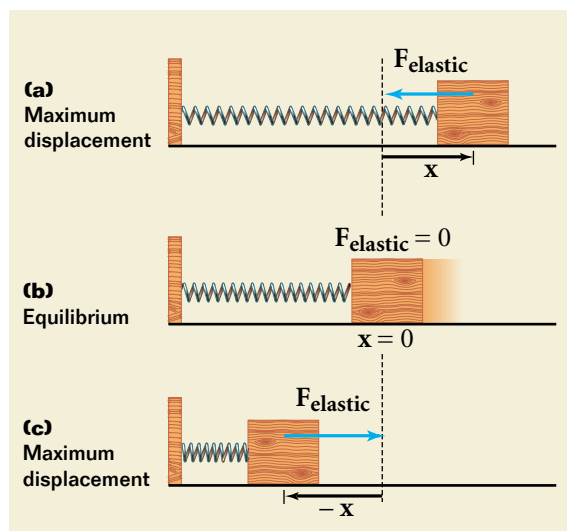
## SECTION OBJECTIVES

- Identify the conditions of simple harmonic motion.
- Explain how force, velocity, and acceleration change as an object vibrates with simple harmonic motion.
- Calculate the spring force using Hooke's law.

## HOOKE'S LAW

A repeated motion, such as that of an acrobat swinging on a trapeze, is called a periodic motion. Other periodic motions include those made by a child on a playground swing, a wrecking ball swaying to and fro, and the pendulum of a grandfather clock or a metronome. In each of these cases, the periodic motion is back and forth over the same path.

One of the simplest types of back-and-forth periodic motion is the motion of a mass attached to a spring, as shown in **Figure 1**. Let us assume that the mass moves on a frictionless horizontal surface. When the spring is stretched or compressed and then released, it vibrates back and forth about its unstretched position. We will begin by considering this example, and then we will apply our conclusions to the swinging motion of a trapeze acrobat.



**Figure 1**

The direction of the force acting on the mass ( $F_{\text{elastic}}$ ) is always opposite the direction of the mass's displacement from equilibrium ( $x = 0$ ). **(a)** When the spring is stretched to the right, the spring force pulls the mass to the left. **(b)** When the spring is unstretched, the spring force is zero. **(c)** When the spring is compressed to the left, the spring force is directed to the right.

## At the equilibrium position, speed reaches a maximum

In **Figure 1(a)**, the spring is stretched away from its unstretched, or equilibrium, position ( $x = 0$ ). In this stretched position, the spring exerts a force on the mass toward the equilibrium position. This spring force decreases as the spring moves toward the equilibrium position, and it reaches zero at equilibrium, as illustrated in **Figure 1(b)**. The mass's acceleration also becomes zero at equilibrium.

Though the spring force and acceleration decrease as the mass moves toward the equilibrium position, the speed of the mass increases. At the equilibrium position, when acceleration reaches zero, the speed reaches a maximum. At that point, although the spring force is zero, the mass's momentum causes it to overshoot the equilibrium position and compress the spring.

## At maximum displacement, spring force and acceleration reach a maximum

As the mass moves beyond equilibrium, the spring force and the acceleration increase. But the direction of the spring force and of the acceleration (toward equilibrium) is opposite the mass's direction of motion (away from equilibrium), and the mass begins to slow down.

When the spring's compression is equal to the distance the spring was originally stretched away from the equilibrium position ( $x$ ), as shown in **Figure 1(c)**, the mass is at maximum displacement, and the spring force and acceleration of the mass reach a maximum. At this point, the speed of the mass becomes zero. The spring force acting to the right causes the mass to change its direction, and the mass begins moving back toward the equilibrium position. Then the entire process begins again, and the mass continues to oscillate back and forth over the same path.

In an ideal system, the mass-spring system would oscillate indefinitely. But in the physical world, friction retards the motion of the vibrating mass, and the mass-spring system eventually comes to rest. This effect is called *damping*. In most cases, the effect of damping is minimal over a short period of time, so the ideal mass-spring system provides an approximation for the motion of a physical mass-spring system.

### In simple harmonic motion, restoring force is proportional to displacement

As you have seen, the spring force always pushes or pulls the mass toward its original equilibrium position. For this reason, it is sometimes called a *restoring force*. Measurements show that the restoring force is directly proportional to the displacement of the mass. This relationship was determined in 1678 by Robert Hooke and is known as *Hooke's Law*. The following equation mathematically describes Hooke's Law:

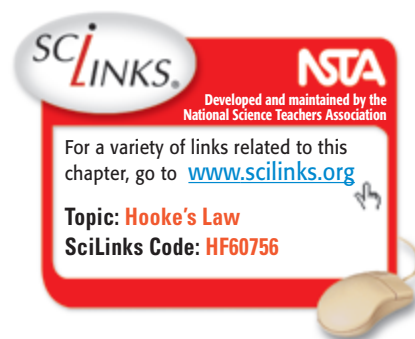
#### HOOKE'S LAW

$$F_{\text{elastic}} = -kx$$

spring force = -(spring constant  $\times$  displacement)

The negative sign in the equation signifies that the direction of the spring force is always opposite the direction of the mass's displacement from equilibrium. In other words, the negative sign shows that the spring force will tend to move the object back to its equilibrium position.

As mentioned in the chapter "Work and Energy," the quantity  $k$  is a positive constant called the *spring constant*. The value of the spring constant is a measure of the stiffness of the spring. A greater value of  $k$  means a stiffer spring because a greater force is needed to stretch or compress that spring a given amount. The SI units of  $k$  are N/m. As a result, N is the unit of the spring force when the spring constant (N/m) is multiplied by the displacement (m). The motion of a vibrating mass-spring system is an example of **simple harmonic motion**. Simple harmonic motion describes any periodic motion that is the result of a restoring force that is proportional to displacement. Because simple harmonic motion involves a restoring force, every simple harmonic motion is a back-and-forth motion over the same path.



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For a variety of links related to this chapter, go to [www.scilinks.org](http://www.scilinks.org)

Topic: **Hooke's Law**  
SciLinks Code: **HF60756**

### simple harmonic motion

vibration about an equilibrium position in which a restoring force is proportional to the displacement from equilibrium



## Conceptual Challenge

#### 1. Earth's Orbit

The motion of Earth orbiting the sun is periodic. Is this motion simple harmonic? Why or why not?

#### 2. Pinball

In pinball games, the force exerted by a compressed spring is used to release a ball. If the distance the spring is compressed is doubled, how will the force exerted on the ball change? If the spring is replaced with one that is half as stiff, how will the force acting on the ball change?

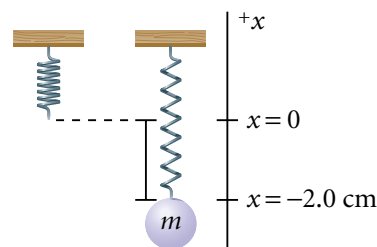


## SAMPLE PROBLEM A

### Hooke's Law

#### PROBLEM

If a mass of 0.55 kg attached to a vertical spring stretches the spring 2.0 cm from its original equilibrium position, what is the spring constant?



#### SOLUTION

##### 1. DEFINE

**Given:**  $m = 0.55 \text{ kg}$   $x = -2.0 \text{ cm} = -0.020 \text{ m}$   
 $g = 9.81 \text{ m/s}^2$

**Unknown:**  $k = ?$

**Diagram:**



##### 2. PLAN

**Choose an equation or situation:** When the mass is attached to the spring, the equilibrium position changes. At the new equilibrium position, the net force acting on the mass is zero. So the spring force (given by Hooke's law) must be equal and opposite to the weight of the mass.

$$\mathbf{F_{net} = 0 = F_{elastic} + F_g}$$

$$F_{elastic} = -kx$$

$$F_g = -mg$$

$$-kx - mg = 0$$

**Rearrange the equation to isolate the unknown:**

$$kx = -mg$$

$$k = \frac{-mg}{x}$$

##### 3. CALCULATE

**Substitute the values into the equation and solve:**

$$k = \frac{-(0.55 \text{ kg})(9.81 \text{ m/s}^2)}{-0.020 \text{ m}}$$

$$k = 270 \text{ N/m}$$

##### 4. EVALUATE

The value of  $k$  implies that 270 N of force is required to displace the spring 1 m.

#### CALCULATOR SOLUTION

The calculator answer for  $k$  is 269.775. This answer is rounded to two significant figures, 270 N/m.

## PRACTICE A

### Hooke's Law

1. Suppose the spring in Sample Problem A is replaced with a spring that stretches 36 cm from its equilibrium position.
  - a. What is the spring constant in this case?
  - b. Is this spring stiffer or less stiff than the one in Sample Problem A?
2. A load of 45 N attached to a spring that is hanging vertically stretches the spring 0.14 m. What is the spring constant?
3. A slingshot consists of a light leather cup attached between two rubber bands. If it takes a force of 32 N to stretch the bands 1.2 cm, what is the equivalent spring constant of the two rubber bands?
4. How much force is required to pull a spring 3.0 cm from its equilibrium position if the spring constant is  $2.7 \times 10^3$  N/m?

### A stretched or compressed spring has elastic potential energy

As you saw in the chapter “Work and Energy,” a stretched or compressed spring stores elastic potential energy. To see how mechanical energy is conserved in an ideal mass-spring system, consider an archer shooting an arrow from a bow, as shown in **Figure 2**. Bending the bow by pulling back the bowstring is analogous to stretching a spring. To simplify this situation, we will disregard friction and internal energy.

Once the bowstring has been pulled back, the bow stores elastic potential energy. Because the bow, arrow, and bowstring (the system) are now at rest, the kinetic energy of the system is zero, and the mechanical energy of the system is solely elastic potential energy.

When the bowstring is released, the bow's elastic potential energy is converted to the kinetic energy of the arrow. At the moment the arrow leaves the bowstring, it gains most of the elastic potential energy originally stored in the bow. (The rest of the elastic potential energy is converted to the kinetic energy of the bow and the bowstring.) Thus, once the arrow leaves the bowstring, the mechanical energy of the bow-and-arrow system is solely kinetic. Because mechanical energy must be conserved, the total kinetic energy of the bow, arrow, and bowstring is equal to the elastic potential energy originally stored in the bow.



**Figure 2**

The elastic potential energy stored in this stretched bow is converted into the kinetic energy of the arrow.



#### Module 11

“Hooke's Law” provides an interactive lesson with guided problem-solving practice to teach you about springs and spring constants.

## THE INSIDE STORY ON SHOCK ABSORBERS

**B**umps in the road are certainly a nuisance, but without strategic use of damping devices, they could also prove deadly. To control a car going 110 km/h (70 mi/h), a driver needs all the wheels on the ground. Bumps in the road lift the wheels off the ground and rob the driver of control. A good solution is to fit the car with springs at each wheel. The springs absorb



energy as the wheels rise over the bumps and push the wheels back to the pavement to keep the wheels on the road. However, once set in motion, springs tend to continue to go up and down in simple

harmonic motion. This affects the driver's control of the car and can also be uncomfortable.

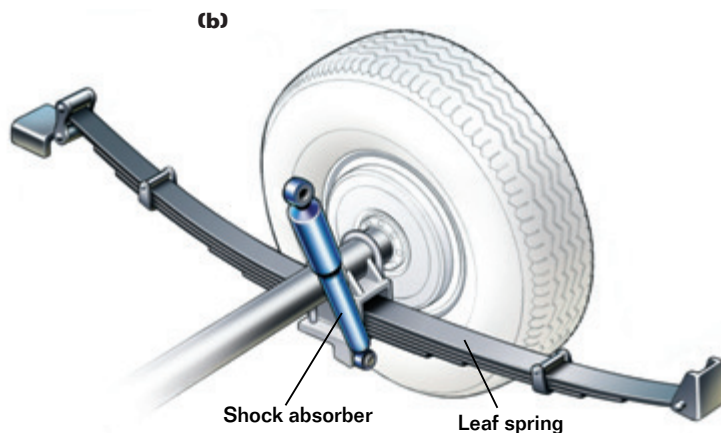
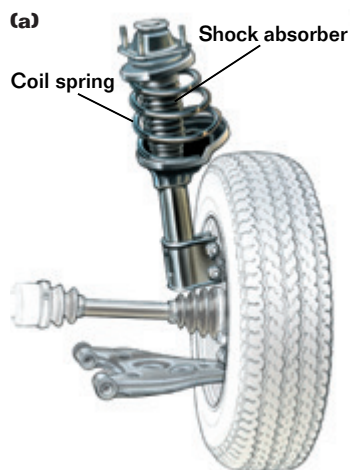
One way to cut down on unwanted vibrations is to use stiff springs that compress only a few centimeters under thousands of newtons of force. Such springs have very high spring constants and thus do not vibrate as freely as softer springs with lower constants. However,

this solution reduces the driver's ability to keep the car's wheels on the road.

To completely solve the problem, energy-absorbing devices known as *shock absorbers* are placed parallel to the springs in some automobiles, as shown in part (a) of the illustration below. Shock absorbers are fluid-filled tubes that turn the simple harmonic motion of the springs into damped harmonic motion. In damped harmonic motion, each cycle of stretch and compression of the spring is much smaller than the previous cycle. Modern auto suspensions are set up so that all of a spring's energy is absorbed by the shock absorbers, eliminating vibrations in just one up-and-down cycle. This keeps the car from continually bouncing without sacrificing the spring's ability to keep the wheels on the road.

Different spring constants and shock absorber damping are combined to give a wide variety of road responses. For example, larger vehicles have heavy-duty leaf springs made of stacks of steel strips, which have a larger spring constant than coil springs do. In this type of suspension system, the shock absorber is perpendicular to the spring, as shown in part (b) of the illustration. The stiffness of the spring can affect steering response time, traction, and the general feel of the car.

As a result of the variety of combinations that are possible, your driving experiences can range from the luxurious floating of a limousine to the bone-rattling road feel of a sports car.

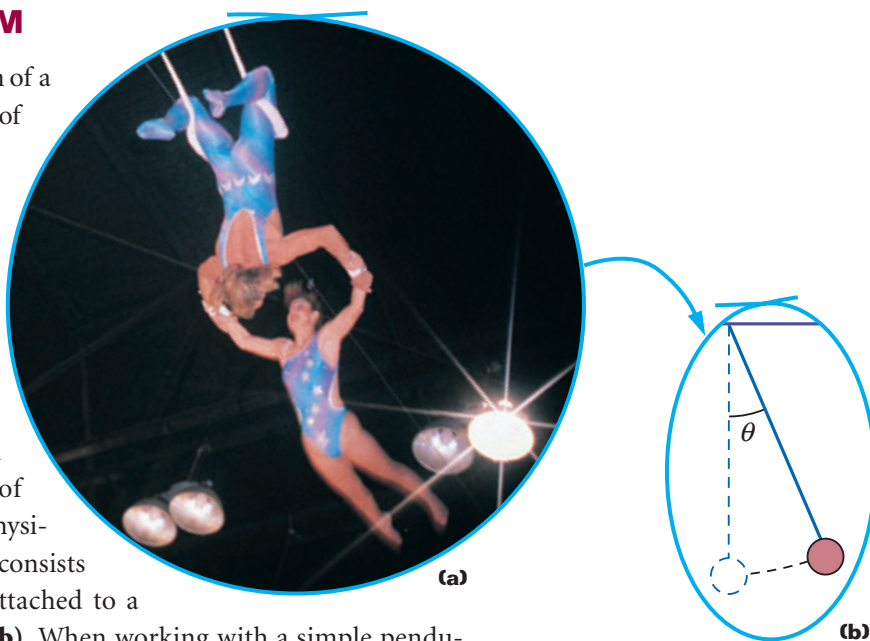




## THE SIMPLE PENDULUM

As you have seen, the periodic motion of a mass-spring system is one example of simple harmonic motion. Now consider the trapeze acrobats shown in **Figure 3(a)**. Like the vibrating mass-spring system, the swinging motion of a trapeze acrobat is a periodic vibration. Is a trapeze acrobat's motion an example of simple harmonic motion?

To answer this question, we will use a simple pendulum as a model of the acrobat's motion, which is a physical pendulum. A simple pendulum consists of a mass called a *bob*, which is attached to a fixed string, as shown in **Figure 3(b)**. When working with a simple pendulum, we assume that the mass of the bob is concentrated at a point and that the mass of the string is negligible. Furthermore, we disregard the effects of friction and air resistance. For a physical pendulum, on the other hand, the distribution of the mass must be considered, and friction and air resistance also must be taken into account. To simplify our analysis, we will disregard these complications and use a simple pendulum to approximate a physical pendulum in all of our examples.

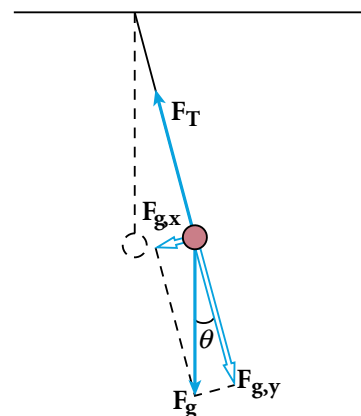


**Figure 3**  
**(a)** The motion of these trapeze acrobats is modeled by **(b)** a simple pendulum.

### The restoring force of a pendulum is a component of the bob's weight

To see whether the pendulum's motion is simple harmonic, we must first examine the forces exerted on the pendulum's bob to determine which force acts as the restoring force. If the restoring force is proportional to the displacement, then the pendulum's motion is simple harmonic. Let us select a coordinate system in which the  $x$ -axis is tangent to the direction of motion and the  $y$ -axis is perpendicular to the direction of motion. Because the bob is always changing its position, these axes will change at each point of the bob's motion.

The forces acting on the bob at any point include the force exerted by the string and the gravitational force. The force exerted by the string always acts along the  $y$ -axis, which is along the string. The gravitational force can be resolved into two components along the chosen axes, as shown in **Figure 4**. Because both the force exerted by the string and the  $y$  component of the gravitational force are perpendicular to the bob's motion, the  $x$  component of the gravitational force is the net force acting on the bob in the direction of its motion. In this case, the  $x$  component of the gravitational force always pulls the bob toward its equilibrium position and hence is the restoring force. Note that the restoring force ( $F_{g,x} = F_g \sin \theta$ ) is zero at equilibrium because  $\theta$  equals zero at this point.



**Figure 4**  
At any displacement from equilibrium, the weight of the bob ( $F_g$ ) can be resolved into two components. The  $x$  component ( $F_{g,x}$ ), which is perpendicular to the string, is the only force acting on the bob in the direction of its motion.



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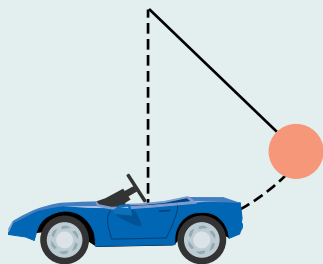
**Topic: Pendulums**  
**SciLinks Code: HF61121**

## Quick Lab

### Energy of a Pendulum

#### MATERIALS LIST

- pendulum bob and string
- tape
- toy car
- protractor
- meterstick or tape measure



Tie one end of a piece of string around the pendulum bob, and use tape to secure it in place. Set the toy car on a smooth surface, and hold the string of the pendulum directly above the car so that the bob rests on the car. Use your other hand to pull back the bob of the pendulum, and have your partner measure the angle of the pendulum with the protractor.

Release the pendulum so that the bob strikes the car. Measure the displacement of the car. What happened to the pendulum's potential energy after you released the bob? Repeat the process using different angles. How can you account for your results?

### For small angles, the pendulum's motion is simple harmonic

As with a mass-spring system, the restoring force of a simple pendulum is not constant. Instead, the magnitude of the restoring force varies with the bob's distance from the equilibrium position. The magnitude of the restoring force is proportional to  $\sin \theta$ . When the maximum angle of displacement  $\theta$  is relatively small ( $<15^\circ$ ),  $\sin \theta$  is approximately equal to  $\theta$  in radians. As a result, the restoring force is very nearly proportional to the displacement and the pendulum's motion is an excellent approximation of simple harmonic motion. We will assume small angles of displacement unless otherwise noted.

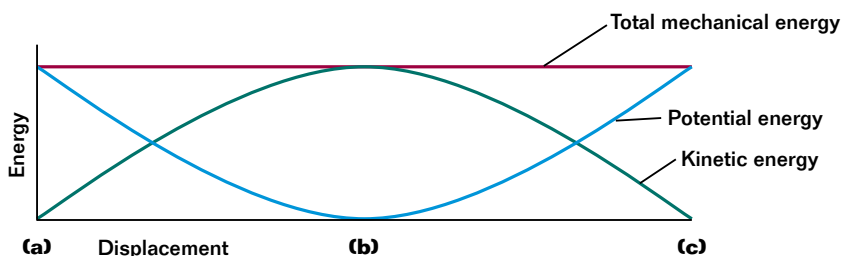
Because a simple pendulum vibrates with simple harmonic motion, many of our earlier conclusions for a mass-spring system apply here. At maximum displacement, the restoring force and acceleration reach a maximum while the speed becomes zero. Conversely, at equilibrium, the restoring force and acceleration become zero and speed reaches a maximum. **Table 1** on the following page illustrates the analogy between a simple pendulum and a mass-spring system.

### Gravitational potential increases as a pendulum's displacement increases

As with the mass-spring system, the mechanical energy of a simple pendulum is conserved in an ideal (frictionless) system. However, the spring's potential energy is elastic, while the pendulum's potential energy is gravitational. We define the gravitational potential energy of a pendulum to be zero when it is at the lowest point of its swing.

**Figure 5** illustrates how a pendulum's mechanical energy changes as the pendulum oscillates. At maximum displacement from equilibrium, a pendulum's energy is entirely gravitational potential energy. As the pendulum swings toward equilibrium, it gains kinetic energy and loses potential energy. At the equilibrium position, its energy becomes solely kinetic.

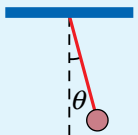
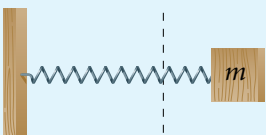
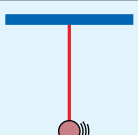
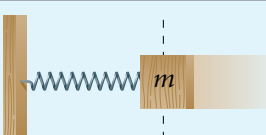
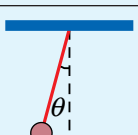

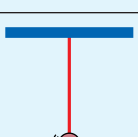
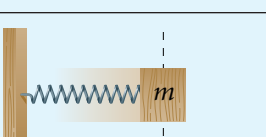
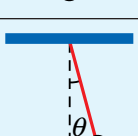
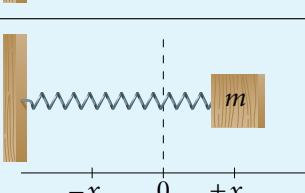
As the pendulum swings past its equilibrium position, the kinetic energy decreases while the gravitational potential energy increases. At maximum displacement from equilibrium, the pendulum's energy is once again entirely gravitational potential energy.



**Figure 5**

Whether at maximum displacement **(a)**, equilibrium **(b)**, or maximum displacement in the other direction **(c)**, the pendulum's total mechanical energy remains the same. However, as the graph shows, the pendulum's kinetic energy and potential energy are constantly changing.

**Table 1 Simple Harmonic Motion**

maximum displacement			$F_x = F_{\max}$ $a = a_{\max}$ $v = 0$
equilibrium			$F_x = 0$ $a = 0$ $v = v_{\max}$
maximum displacement			$F_x = F_{\max}$ $a = a_{\max}$ $v = 0$
equilibrium			$F_x = 0$ $a = 0$ $v = v_{\max}$
maximum displacement			$F_x = F_{\max}$ $a = a_{\max}$ $v = 0$

## SECTION REVIEW

- Which of these periodic motions are simple harmonic?
  - a child swinging on a playground swing ( $\theta = 45^\circ$ )
  - a CD rotating in a player
  - an oscillating clock pendulum ( $\theta = 10^\circ$ )
- A pinball machine uses a spring that is compressed 4.0 cm to launch a ball. If the spring constant is 13 N/m, what is the force on the ball at the moment the spring is released?
- How does the restoring force acting on a pendulum bob change as the bob swings toward the equilibrium position? How do the bob's acceleration (along the direction of motion) and velocity change?
- Critical Thinking** When an acrobat reaches the equilibrium position, the net force acting along the direction of motion is zero. Why does the acrobat swing past the equilibrium position?

## SECTION 2

# Measuring Simple Harmonic Motion

### SECTION OBJECTIVES

- Identify the amplitude of vibration.
- Recognize the relationship between period and frequency.
- Calculate the period and frequency of an object vibrating with simple harmonic motion.

#### amplitude

the maximum displacement from equilibrium

#### period

the time that it takes a complete cycle to occur

#### frequency

the number of cycles or vibrations per unit of time

## AMPLITUDE, PERIOD, AND FREQUENCY

In the absence of friction, a moving trapeze always returns to the same maximum displacement after each swing. This maximum displacement from the equilibrium position is the **amplitude**. A pendulum's amplitude can be measured by the angle between the pendulum's equilibrium position and its maximum displacement. For a mass-spring system, the amplitude is the maximum amount the spring is stretched or compressed from its equilibrium position.

### Period and frequency measure time

Imagine the ride shown in **Figure 6** swinging from maximum displacement on one side of equilibrium to maximum displacement on the other side, and then back again. This cycle is considered one complete cycle of motion. The **period**,  $T$ , is the time it takes for this complete cycle of motion. For example, if one complete cycle takes 20 s, then the period of this motion is 20 s. Note that after the time  $T$ , the object is back where it started.

The number of complete cycles the ride swings through in a unit of time is the ride's **frequency**,  $f$ . If one complete cycle takes 20 s, then the ride's frequency is  $\frac{1}{20}$  cycles/s, or 0.05 cycles/s. The SI unit of frequency is  $\text{s}^{-1}$ , known as hertz (Hz). In this case, the ride's frequency is 0.05 Hz.

Period and frequency can be confusing because both are concepts involving time in simple harmonic motion. Notice that the period is the time per cycle and that the frequency is the number of cycles per unit time, so they are inversely related.

$$f = \frac{1}{T} \text{ or } T = \frac{1}{f}$$

This relationship was used to determine the frequency of the ride.

$$f = \frac{1}{T} = \frac{1}{20 \text{ s}} = 0.05 \text{ Hz}$$

In any problem where you have a value for period or frequency, you can calculate the other value. These terms are summarized in **Table 2** on the next page.

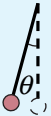
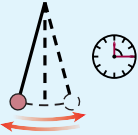
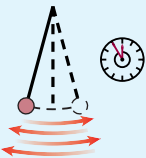
#### Figure 6

For any periodic motion—such as the motion of this amusement park ride in Helsinki, Finland—period and frequency are inversely related.





**Table 2 Measures of Simple Harmonic Motion**

Term	Example	Definition	SI unit
amplitude		maximum displacement from equilibrium	radian, rad meter, m
period, $T$		time that it takes to complete a full cycle	second, s
frequency, $f$		number of cycles or vibrations per unit of time	hertz, Hz (Hz = s <sup>-1</sup> )

**extension****Integrating Technology**

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**Keyword HF6VIBX**

### The period of a simple pendulum depends on pendulum length and free-fall acceleration

Although both a simple pendulum and a mass-spring system vibrate with simple harmonic motion, calculating the period and frequency of each requires a separate equation. This is because in each, the period and frequency depend on different physical factors.

Consider an experimental setup of two pendulums of the same length but with bobs of different masses. The length of a pendulum is measured from the pivot point to the center of mass of the pendulum bob. If you were to pull each bob aside the same small distance and then release them at the same time, each pendulum would complete one vibration in the same amount of time. If you then changed the amplitude of one of the pendulums, you would find that they would still have the same period. Thus, for small amplitudes, the period of a pendulum does not depend on the mass or on the amplitude.

However, changing the length of a pendulum *does* affect its period. A change in the free-fall acceleration also affects the period of a pendulum. The exact relationship between these variables can be derived mathematically or found experimentally.

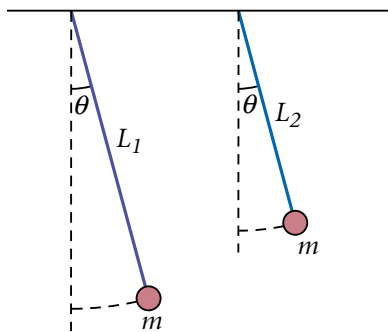
#### PERIOD OF A SIMPLE PENDULUM IN SIMPLE HARMONIC MOTION

$$T = 2\pi \sqrt{\frac{L}{a_g}}$$

period =  $2\pi \times$  square root of (length divided by free-fall acceleration)

### Did you know?

Galileo is credited as the first person to notice that the motion of a pendulum depends on its length and is independent of its amplitude (for small angles). He supposedly observed this while attending church services at a cathedral in Pisa. The pendulum he studied was a swinging chandelier that was set in motion when someone bumped it while lighting the candles. Galileo is said to have measured its frequency, and hence its period, by timing the swings with his pulse.



**Figure 7**

When the length of one pendulum is decreased, the distance that the pendulum travels to equilibrium is also decreased. Because the accelerations of the two pendulums are equal, the shorter pendulum will have a smaller period.

Why does the period of a pendulum depend on pendulum length and free-fall acceleration? When two pendulums have different lengths but the same amplitude, the shorter pendulum will have a smaller arc to travel through, as shown in **Figure 7**. Because the distance from maximum displacement to equilibrium is less while the acceleration caused by the restoring force remains the same, the shorter pendulum will have a shorter period.

Why don't mass and amplitude affect the period of a pendulum? When the bobs of two pendulums differ in mass, the heavier mass provides a larger restoring force, but it also needs a larger force to achieve the same acceleration. This is similar to the situation for objects in free fall, which all have the same acceleration regardless of their mass. Because the acceleration of both pendulums is the same, the period for both is also the same.

For small angles ( $<15^\circ$ ), when the amplitude of a pendulum increases, the restoring force also increases proportionally. Because force is proportional to acceleration, the initial acceleration will be greater. However, the distance this pendulum must cover is also greater. For small angles, the effects of the two increasing quantities cancel and the pendulum's period remains the same.

## SAMPLE PROBLEM B

### Simple Harmonic Motion of a Simple Pendulum

#### PROBLEM

You need to know the height of a tower, but darkness obscures the ceiling. You note that a pendulum extending from the ceiling almost touches the floor and that its period is 12 s. How tall is the tower?

#### SOLUTION

**Given:**  $T = 12 \text{ s}$       $a_g = g = 9.81 \text{ m/s}^2$

**Unknown:**  $L = ?$

Use the equation for the period of a simple pendulum, and solve for  $L$ .

$$T = 2\pi \sqrt{\frac{L}{a_g}}$$

$$\frac{T\sqrt{a_g}}{2\pi} = \sqrt{L}$$

$$\frac{T^2 a_g}{4\pi^2} = L$$

$$L = \frac{(12 \text{ s})^2 (9.81 \text{ m/s}^2)}{4\pi^2}$$

$$L = 36 \text{ m}$$

**TIP**

Remember that on Earth's surface,  $a_g = g = 9.81 \text{ m/s}^2$ . Use this value in the equation for the period of a pendulum if a problem does not specify otherwise. At higher altitudes or on different planets, use the given value of  $a_g$  instead.

## PRACTICE B

### Simple Harmonic Motion of a Simple Pendulum

1. If the period of the pendulum in the preceding sample problem were 24 s, how tall would the tower be?
2. You are designing a pendulum clock to have a period of 1.0 s. How long should the pendulum be?
3. A trapeze artist swings in simple harmonic motion with a period of 3.8 s. Calculate the length of the cables supporting the trapeze.
4. Calculate the period and frequency of a 3.500 m long pendulum at the following locations:
  - a. the North Pole, where  $a_g = 9.832 \text{ m/s}^2$
  - b. Chicago, where  $a_g = 9.803 \text{ m/s}^2$
  - c. Jakarta, Indonesia, where  $a_g = 9.782 \text{ m/s}^2$

### Period of a mass-spring system depends on mass and spring constant

Now consider the period of a mass-spring system. In this case, according to Hooke's law, the restoring force acting on the mass is determined by the displacement of the mass and by the spring constant ( $F_{\text{elastic}} = -kx$ ). The magnitude of the mass does not affect the restoring force. So, unlike in the case of the pendulum, in which a heavier mass increased both the force on the bob and the bob's inertia, a heavier mass attached to a spring increases inertia without providing a compensating increase in force. Because of this increase in inertia, a heavy mass has a smaller acceleration than a light mass has. Thus, a heavy mass will take more time to complete one cycle of motion. In other words, the heavy mass has a greater period. Thus, as mass increases, the period of vibration increases when there is no compensating increase in force.

## Conceptual Challenge



### 1. Pendulum on the Moon

The free-fall acceleration on the surface of the moon is approximately one-sixth of the free-fall acceleration on the surface of Earth. Compare the period of a pendulum on the moon with that of an identical pendulum set in motion on Earth.

### 2. Pendulum Clocks

Why is a wound mainspring often used to provide energy to a pendulum clock in order to prevent the amplitude of the pendulum from decreasing?





The greater the spring constant ( $k$ ), the stiffer the spring; hence a greater force is required to stretch or compress the spring. When force is greater, acceleration is greater and the amount of time required for a single cycle should decrease (assuming that the amplitude remains constant). Thus, for a given amplitude, a stiffer spring will take less time to complete one cycle of motion than one that is less stiff.

As with the pendulum, the equation for the period of a mass-spring system can be derived mathematically or found experimentally.

#### PERIOD OF A MASS-SPRING SYSTEM IN SIMPLE HARMONIC MOTION

$$T = 2\pi \sqrt{\frac{m}{k}}$$

period =  $2\pi \times$  square root of (mass divided by spring constant)

Note that changing the amplitude of the vibration does not affect the period. This statement is true only for systems and circumstances in which the spring obeys Hooke's law.

### SAMPLE PROBLEM C

#### Simple Harmonic Motion of a Mass-Spring System

##### PROBLEM

The body of a 1275 kg car is supported on a frame by four springs. Two people riding in the car have a combined mass of 153 kg. When driven over a pothole in the road, the frame vibrates with a period of 0.840 s. For the first few seconds, the vibration approximates simple harmonic motion. Find the spring constant of a single spring.

##### SOLUTION

**Given:**  $m = \frac{(1275 \text{ kg} + 153 \text{ kg})}{4} = 357 \text{ kg}$        $T = 0.840 \text{ s}$

**Unknown:**  $k = ?$

Use the equation for the period of a mass-spring system, and solve for  $k$ .

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$T^2 = 4\pi^2 \left( \frac{m}{k} \right)$$

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 (357 \text{ kg})}{(0.840 \text{ s})^2}$$

$$k = 2.00 \times 10^4 \text{ N/m}$$

## PRACTICE C

### Simple Harmonic Motion of a Mass-Spring System

1. A mass of 0.30 kg is attached to a spring and is set into vibration with a period of 0.24 s. What is the spring constant of the spring?
2. When a mass of 25 g is attached to a certain spring, it makes 20 complete vibrations in 4.0 s. What is the spring constant of the spring?
3. A 125 N object vibrates with a period of 3.56 s when hanging from a spring. What is the spring constant of the spring?
4. When two more people get into the car described in Sample Problem C, the total mass of all four occupants of the car becomes 255 kg. Now what is the period of vibration of the car when it is driven over a pothole in the road?
5. A spring of spring constant 30.0 N/m is attached to different masses, and the system is set in motion. Find the period and frequency of vibration for masses of the following magnitudes:
  - a. 2.3 kg
  - b. 15 g
  - c. 1.9 kg

## SECTION REVIEW

1. The reading on a metronome indicates the number of oscillations per minute. What are the frequency and period of the metronome's vibration when the metronome is set at 180?
2. A child swings on a playground swing with a 2.5 m long chain.
  - a. What is the period of the child's motion?
  - b. What is the frequency of vibration?
3. A 0.75 kg mass attached to a vertical spring stretches the spring 0.30 m.
  - a. What is the spring constant?
  - b. The mass-spring system is now placed on a horizontal surface and set vibrating. What is the period of the vibration?
4. **Critical Thinking** Two mass-spring systems vibrate with simple harmonic motion. If the spring constants are equal and the mass of one system is twice that of the other, which system has a greater period?

## SECTION OBJECTIVES

- Distinguish local particle vibrations from overall wave motion.
- Differentiate between pulse waves and periodic waves.
- Interpret waveforms of transverse and longitudinal waves.
- Apply the relationship among wave speed, frequency, and wavelength to solve problems.
- Relate energy and amplitude.

**Figure 8**

A pebble dropped into a pond creates ripple waves similar to those shown here.

## WAVE MOTION

Consider what happens to the surface of a pond when you drop a pebble into the water. The disturbance created by the pebble generates water waves that travel away from the disturbance, as seen in **Figure 8**. If you examined the motion of a leaf floating near the disturbance, you would see that the leaf moves up and down and back and forth about its original position. However, the leaf does not undergo any net displacement from the motion of the waves.

The leaf's motion indicates the motion of the particles in the water. The water molecules move locally, like the leaf does, but they do not travel across the pond. That is, the water wave moves from one place to another, but the water itself is not carried with it.



### A wave is the motion of a disturbance

Ripple waves in a pond start with a disturbance at some point in the water. This disturbance causes water on the surface near that point to move, which in turn causes points farther away to move. In this way, the waves travel outward in a circular pattern away from the original disturbance.

In this example, the water in the pond is the **medium** through which the disturbance travels. Particles in the medium—in this case, water molecules—move in vertical circles as waves pass. Note that the medium does not actually travel with the waves. After the waves have passed, the water returns to its original position.

Waves of almost every kind require a material medium in which to travel. Sound waves, for example, cannot travel through outer space, because space is very nearly a vacuum. In order for sound waves to travel, they must have a medium such as air or water. Waves that require a material medium are called **mechanical waves**.

Not all wave propagation requires a medium. Electromagnetic waves, such as visible light, radio waves, microwaves, and X rays, can travel through a vacuum. You will study electromagnetic waves in later chapters.

### medium

*a physical environment through which a disturbance can travel*

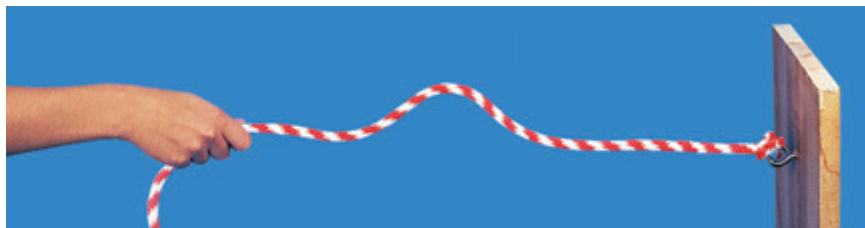
### mechanical wave

*a wave that requires a medium through which to travel*



## WAVE TYPES

One of the simplest ways to demonstrate wave motion is to flip one end of a taut rope whose opposite end is fixed, as shown in **Figure 9**. The flip of your wrist creates a pulse that travels to the fixed end with a definite speed. A wave that consists of a single traveling pulse is called a *pulse wave*.



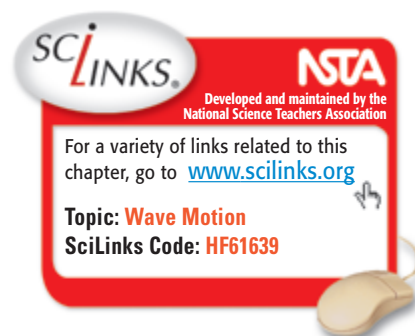
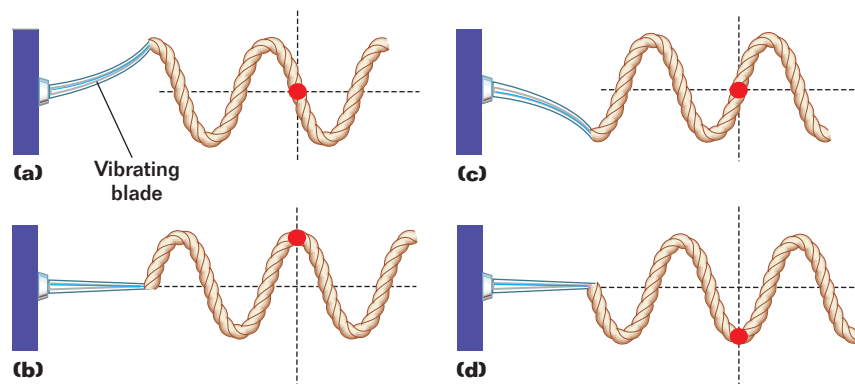
**Figure 9**  
A single flip of the wrist creates a pulse wave on a taut rope.

Now imagine that you continue to generate pulses at one end of the rope. Together, these pulses form what is called a *periodic wave*. Whenever the source of a wave's motion is a periodic motion, such as the motion of your hand moving up and down repeatedly, a periodic wave is produced.

### Sine waves describe particles vibrating with simple harmonic motion

**Figure 10** depicts a blade that vibrates with simple harmonic motion and thus makes a periodic wave on a string. As the wave travels to the right, any single point on the string vibrates up and down. Because the blade is vibrating with simple harmonic motion, the vibration of each point of the string is also simple harmonic. A wave whose source vibrates with simple harmonic motion is called a *sine wave*. Thus, a sine wave is a special case of a periodic wave in which the periodic motion is simple harmonic. The wave in **Figure 10** is called a sine wave because a graph of the trigonometric function  $y = \sin x$  produces this curve when plotted.

A close look at a single point on the string illustrated in **Figure 10** shows that its motion resembles the motion of a mass hanging from a vibrating spring. As the wave travels to the right, the point vibrates around its equilibrium position with simple harmonic motion. This relationship between simple harmonic motion and wave motion enables us to use some of the terms and concepts from simple harmonic motion in our study of wave motion.



**Figure 10**  
As the sine wave created by this vibrating blade travels to the right, a single point on the string vibrates up and down with simple harmonic motion.

### transverse wave

a wave whose particles vibrate perpendicularly to the direction the wave is traveling

### crest

the highest point above the equilibrium position

### trough

the lowest point below the equilibrium position

### wavelength

the distance between two adjacent similar points of a wave, such as from crest to crest or from trough to trough

## Vibrations of a transverse wave are perpendicular to the wave motion

**Figure 11(a)** is a representation of the wave shown in **Figure 10** (on the previous page) at a specific instant of time,  $t$ . This wave travels to the right as the particles of the rope vibrate up and down. Thus, the vibrations are perpendicular to the direction of the wave's motion. A wave such as this, in which the particles of the disturbed medium move perpendicularly to the wave motion, is called a **transverse wave**.

The wave shown in **Figure 11(a)** can be represented on a coordinate system, as shown in **Figure 11(b)**. A picture of a wave like the one in **Figure 11(b)** is sometimes called a *waveform*. A waveform can represent either the displacements of each point of the wave at a single moment in time or the displacements of a single particle as time passes.

In this case, the waveform depicts the displacements at a single instant. The  $x$ -axis represents the equilibrium position of the string, and the  $y$  coordinates of the curve represent the displacement of each point of the string at time  $t$ . For example, points where the curve crosses the  $x$ -axis (where  $y = 0$ ) have zero displacement. Conversely, at the highest and lowest points of the curve, where displacement is greatest, the absolute values of  $y$  are greatest.

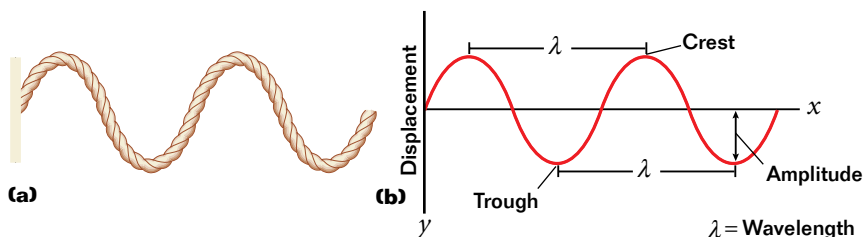
## Wave measures include crest, trough, amplitude, and wavelength

A wave can be measured in terms of its displacement from equilibrium. The highest point above the equilibrium position is called the wave **crest**. The lowest point below the equilibrium position is the **trough** of the wave. As in simple harmonic motion, amplitude is a measure of maximum displacement from equilibrium. The amplitude of a wave is the distance from the equilibrium position to a crest or to a trough, as shown in **Figure 11(b)**.

As a wave passes a given point along its path, that point undergoes cyclical motion. The point is displaced first in one direction and then in the other direction. Finally, the point returns to its original equilibrium position, thereby completing one cycle. The distance the wave travels along its path during one cycle is called the **wavelength**,  $\lambda$  (the Greek letter *lambda*). A simple way to find the wavelength is to measure the distance between two adjacent similar points of the wave, such as from crest to crest or from trough to trough. Notice in **Figure 11(b)** that the distances between adjacent crests or troughs in the waveform are equal.

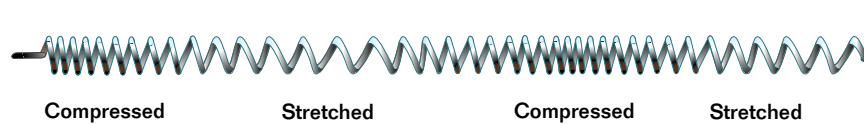
**Figure 11**

(a) A picture of a transverse wave at some instant  $t$  can be turned into (b) a graph. The  $x$ -axis represents the equilibrium position of the string. The curve shows the displacements of the string at time  $t$ .



## Vibrations of a longitudinal wave are parallel to the wave motion

You can create another type of wave with a spring. Suppose that one end of the spring is fixed and that the free end is pumped back and forth along the length of the spring, as shown in **Figure 12**. This action produces compressed and stretched regions of the coil that travel along the spring. The displacement of the coils is in the direction of wave motion. In other words, the vibrations are parallel to the motion of the wave.



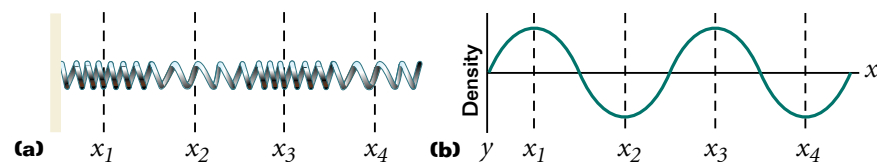
**Figure 12**

As this wave travels to the right, the coils of the spring are tighter in some regions and looser in others. The displacement of the coils is parallel to the direction of wave motion, so this wave is longitudinal.

When the particles of the medium vibrate parallel to the direction of wave motion, the wave is called a **longitudinal wave**. Sound waves in the air are longitudinal waves because air particles vibrate back and forth in a direction parallel to the direction of wave motion.

A longitudinal wave can also be described by a sine curve. Consider a longitudinal wave traveling on a spring. **Figure 13(a)** is a snapshot of the longitudinal wave at some instant  $t$ , and **Figure 13(b)** shows the sine curve representing the wave. The compressed regions correspond to the crests of the waveform, and the stretched regions correspond to troughs.

The type of wave represented by the curve in **Figure 13(b)** is often called a *density wave* or a *pressure wave*. The crests, where the spring coils are compressed, are regions of high density and pressure (relative to the equilibrium density or pressure of the medium). Conversely, the troughs, where the coils are stretched, are regions of low density and pressure.



### longitudinal wave

*a wave whose particles vibrate parallel to the direction the wave is traveling*

**Figure 13**

(a) A longitudinal wave at some instant  $t$  can also be represented by (b) a graph. The crests of this waveform correspond to compressed regions, and the troughs correspond to stretched regions.

## PERIOD, FREQUENCY, AND WAVE SPEED

Sound waves may begin with the vibrations of your vocal cords, a guitar string, or a taut drumhead. In each of these cases, the source of wave motion is a vibrating object. The vibrating object that causes a sine wave always has a characteristic frequency. Because this motion is transferred to the particles of the medium, the frequency of vibration of the particles is equal to the frequency of the source. When the vibrating particles of the medium complete one full cycle, one complete wavelength passes any given point. Thus, wave frequency describes the number of waves that pass a given point in a unit of time.



The period of a wave is the time required for one complete cycle of vibration of the medium's particles. As the particles of the medium complete one full cycle of vibration at any point of the wave, one wavelength passes by that point. Thus, the period of a wave describes the time it takes for a complete wavelength to pass a given point. The relationship between period and frequency seen earlier in this chapter holds true for waves as well; the period of a wave is inversely related to its frequency.

## Did you know?

The frequencies of sound waves that are audible to humans range from 20 Hz to 20 000 Hz. Electromagnetic waves, which include visible light, radio waves, and microwaves, have an even broader range of frequencies—from about  $10^4$  Hz and lower to  $10^{25}$  Hz and higher.

## Wave speed equals frequency times wavelength

We can now derive an expression for the speed of a wave in terms of its period or frequency. We know that speed is equal to displacement divided by the time it takes to undergo that displacement.

$$v = \frac{\Delta x}{\Delta t}$$

For waves, a displacement of one wavelength ( $\lambda$ ) occurs in a time interval equal to one period of the vibration ( $T$ ).

$$v = \frac{\lambda}{T}$$

As you saw earlier in this chapter, frequency and period are inversely related.

$$f = \frac{1}{T}$$

Substituting this frequency relationship into the previous equation for speed gives a new equation for the speed of a wave.

$$v = \frac{\lambda}{T} = f\lambda$$



### Module 12

“Frequency and Wavelength” provides an interactive lesson with guided problem-solving practice to teach you about wave properties and the wave-speed equation.

## SPEED OF A WAVE

$$v = f\lambda$$

**speed of a wave = frequency  $\times$  wavelength**

The speed of a mechanical wave is constant for any given medium. For example, at a concert, sound waves from different instruments reach your ears at the same moment, even when the frequencies of the sound waves are different. Thus, although the frequencies and wavelengths of the sounds produced by each instrument may be different, the product of the frequency and wavelength is always the same at the same temperature. As a result, when a mechanical wave's frequency is increased, its wavelength must decrease in order for its speed to remain constant. The speed of a wave changes only when the wave moves from one medium to another or when certain properties of the medium (such as temperature) are varied.

## SAMPLE PROBLEM D

### Wave Speed

#### PROBLEM

A piano string tuned to middle C vibrates with a frequency of 262 Hz. Assuming the speed of sound in air is 343 m/s, find the wavelength of the sound waves produced by the string.

#### SOLUTION

**Given:**  $v = 343 \text{ m/s}$        $f = 262 \text{ Hz}$

**Unknown:**  $\lambda = ?$

Use the equation relating speed, wavelength, and frequency for a wave.

$$v = f\lambda$$

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{262 \text{ Hz}} = \frac{343 \text{ m} \cdot \text{s}^{-1}}{262 \text{ s}^{-1}}$$

$$\lambda = 1.31 \text{ m}$$

## PRACTICE D

### Wave Speed

1. A piano emits frequencies that range from a low of about 28 Hz to a high of about 4200 Hz. Find the range of wavelengths in air attained by this instrument when the speed of sound in air is 340 m/s.
2. The speed of all electromagnetic waves in empty space is  $3.00 \times 10^8 \text{ m/s}$ . Calculate the wavelength of electromagnetic waves emitted at the following frequencies:
  - a. radio waves at 88.0 MHz
  - b. visible light at  $6.0 \times 10^8 \text{ MHz}$
  - c. X rays at  $3.0 \times 10^{12} \text{ MHz}$
3. The red light emitted by a He-Ne laser has a wavelength of 633 nm in air and travels at  $3.00 \times 10^8 \text{ m/s}$ . Find the frequency of the laser light.
4. A tuning fork produces a sound with a frequency of 256 Hz and a wavelength in air of 1.35 m.
  - a. What value does this give for the speed of sound in air?
  - b. What would be the wavelength of this same sound in water in which sound travels at 1500 m/s?

### extension

#### Integrating Earth Science

Visit [go.hrw.com](http://go.hrw.com) for the activity "Earthquake Waves."



Keyword HF6VIBX

### Waves transfer energy

When a pebble is dropped into a pond, the water wave that is produced carries a certain amount of energy. As the wave spreads to other parts of the pond, the energy likewise moves across the pond. Thus, the wave transfers energy from one place in the pond to another while the water remains in essentially the same place. In other words, waves transfer energy by the vibration of matter rather than by the transfer of matter itself. For this reason, waves are often able to transport energy efficiently.

The rate at which a wave transfers energy depends on the amplitude at which the particles of the medium are vibrating. The greater the amplitude, the more energy a wave carries in a given time interval. For a mechanical wave, the energy transferred is proportional to the square of the wave's amplitude. When the amplitude of a mechanical wave is doubled, the energy it carries in a given time interval increases by a factor of four. Conversely, when the amplitude is halved, the energy decreases by a factor of four.

As with a mass-spring system or a simple pendulum, the amplitude of a wave gradually diminishes over time as its energy is dissipated. This effect, called *damping*, is usually minimal over relatively short distances. For simplicity, we have disregarded damping in our analysis of wave motions.

## SECTION REVIEW

1. As waves pass by a duck floating on a lake, the duck bobs up and down but remains in essentially one place. Explain why the duck is not carried along by the wave motion.
2. Sketch each of the following waves that are on a spring that is attached to a wall at one end:
  - a. a pulse wave that is longitudinal
  - b. a periodic wave that is longitudinal
  - c. a pulse wave that is transverse
  - d. a periodic wave that is transverse
3. Draw a graph for each of the waves described in items (b) and (d) above, and label the  $y$ -axis of each graph with the appropriate variable. Label the following on each graph: crest, trough, wavelength, and amplitude.
4. If the amplitude of a sound wave is increased by a factor of four, how does the energy carried by the sound wave in a given time interval change?
5. The smallest insects that a bat can detect are approximately the size of one wavelength of the sound the bat makes. What is the minimum frequency of sound waves required for the bat to detect an insect that is 0.57 cm long? (Assume the speed of sound is 340 m/s.)

# Wave Interactions

## SECTION 4

### WAVE INTERFERENCE

When two bumper boats collide, as shown in **Figure 14**, each bounces back in another direction. The two bumper boats cannot occupy the same space, and so they are forced to change the direction of their motion. This is true not just of bumper boats but of all matter. Two different material objects can never occupy the same space at the same time.



When two waves come together, they do not bounce back as bumper boats do. If you listen carefully at a concert, you can distinguish the sounds of different instruments. Trumpet sounds are different from flute sounds, even when the two instruments are played at the same time. The sound waves of each instrument are unaffected by the other waves that are passing through the same space at the same moment. Because mechanical waves are not matter but rather are displacements of matter, two waves can occupy the same space at the same time. The combination of two overlapping waves is called *superposition*.

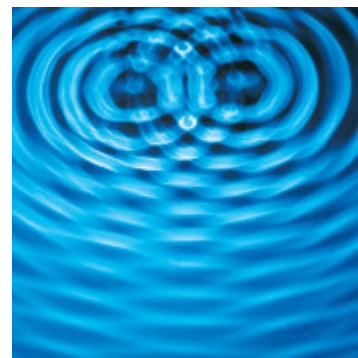
**Figure 15** shows two sets of water waves in a ripple tank. As the waves move outward from their respective sources, they pass through one another. As they pass through one another, the waves interact to form an *interference pattern* of light and dark bands. Although this superposition of mechanical waves is fairly easy to observe, these are not the only kind of waves that can pass through the same space at the same time. Visible light and other forms of electromagnetic radiation also undergo superposition, and they can interact to form interference patterns.

### SECTION OBJECTIVES

- Apply the superposition principle.
- Differentiate between constructive and destructive interference.
- Predict when a reflected wave will be inverted.
- Predict whether specific traveling waves will produce a standing wave.
- Identify nodes and antinodes of a standing wave.

**Figure 14**

Two of these bumper boats cannot be in the same place at one time. Waves, on the other hand, can pass through one another.



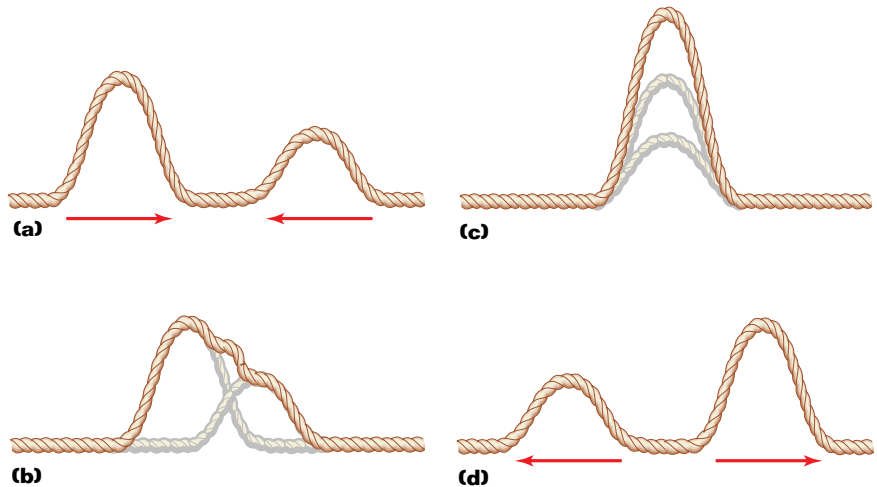
**Figure 15**

This ripple tank demonstrates the interference of water waves.



## Displacements in the same direction produce constructive interference

In **Figure 16(a)**, two wave pulses are traveling toward each other on a stretched rope. The larger pulse is moving to the right, while the smaller pulse moves toward the left. At the moment the two wave pulses meet, a resultant wave is formed, as shown in **Figure 16(b)**.



**Figure 16**

When these two wave pulses meet, the displacements at each point add up to form a resultant wave. This is an example of constructive interference.

At each point along the rope, the displacements due to the two pulses are added together, and the result is the displacement of the resultant wave. For example, when the two pulses exactly coincide, as they do in **Figure 16(c)**, the amplitude of the resultant wave is equal to the sum of the amplitudes of each pulse. This method of summing the displacements of waves is known as the *superposition principle*. According to this principle, when two or more waves travel through a medium at the same time, the resultant wave is the sum of the displacements of the individual waves at each point. Ideally, the superposition principle holds true for all types of waves, both mechanical and electromagnetic. However, experiments show that in reality the superposition principle is valid only when the individual waves have small amplitudes—an assumption we make in all our examples.

Notice that after the two pulses pass through each other, each pulse has the same shape it had before the waves met and each is still traveling in the same direction, as shown in **Figure 16(d)**. This is true for sound waves at a concert, water waves in a pond, light waves, and other types of waves. Each wave maintains its own characteristics after interference, just as the two pulses do in our example above.

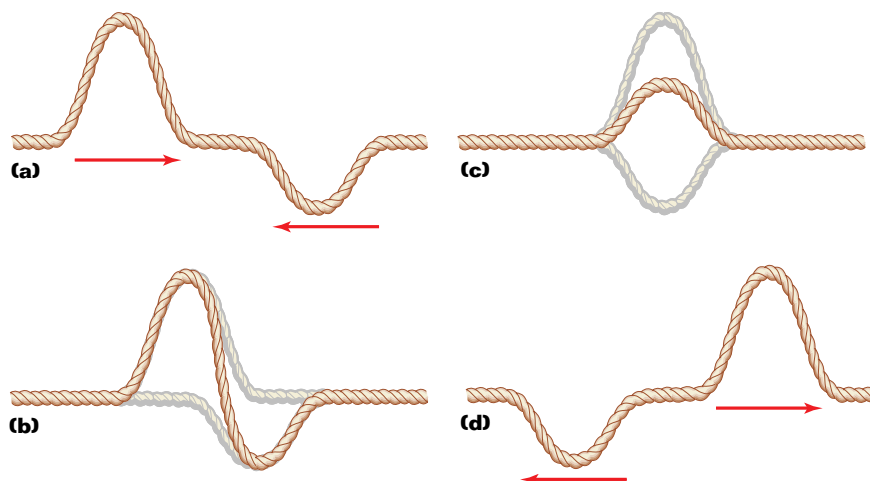
### constructive interference

a superposition of two or more waves in which individual displacements on the same side of the equilibrium position are added together to form the resultant wave

You have seen that when more than one wave travels through the same space at the same time, the resultant wave is equal to the sum of the individual displacements. If the displacements are on the same side of equilibrium, as in **Figure 16**, they have the same sign. When added together, the resultant wave is larger than the individual displacements. This is called **constructive interference**.

## Displacements in opposite directions produce destructive interference

What happens if the pulses are on opposite sides of the equilibrium position, as they are in **Figure 17(a)**? In this case, the displacements have different signs, one positive and one negative. When the positive and negative displacements are added, as shown in **Figure 17(b)** and **(c)**, the resultant wave is the difference between the pulses. This is called **destructive interference**. After the pulses separate, their shapes are unchanged, as seen in **Figure 17(d)**.



### destructive interference

a superposition of two or more waves in which individual displacements on opposite sides of the equilibrium position are added together to form the resultant wave

**Figure 17**

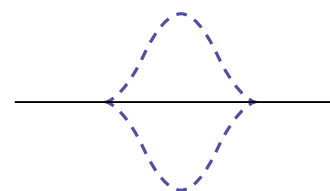
In this case, known as destructive interference, the displacement of one pulse is subtracted from the displacement of the other.

**Figure 18** shows two pulses of equal amplitude but with displacements of opposite signs. When the two pulses coincide and the displacements are added, the resultant wave has a displacement of zero. In other words, at the instant the two pulses overlap, they completely cancel each other; it is as if there were no disturbance at all. This situation is known as *complete destructive interference*.

If these waves were water waves coming together, one of the waves would be acting to pull an individual drop of water upward at the same instant and with the same force that another wave would be acting to pull it downward. The result would be no net force on the drop, and there would be no net displacement of the water at that moment.

Thus far, we have considered the interference produced by two transverse pulse waves. The superposition principle is valid for longitudinal waves as well. In a *compression*, particles are moved closer together, while in a *rarefaction*, particles are spread farther apart. So, when a compression and a rarefaction interfere, there is destructive interference.

In our examples, we have considered constructive and destructive interference separately, and we have dealt only with pulse waves. With periodic waves, complicated patterns arise that involve regions of constructive and destructive interference. The locations of these regions may remain fixed or may vary with time as the individual waves travel.



**Figure 18**

The resultant displacement at each point of the string is zero, so the two pulses cancel one another. This is complete destructive interference.

## ADVANCED TOPICS

See “De Broglie Waves” in **Appendix J: Advanced Topics** to learn about the wave characteristics that all matter exhibits at the microscopic level.

## REFLECTION

In our discussion of waves so far, we have assumed that the waves being analyzed could travel indefinitely without striking anything that would stop them or otherwise change their motion. But what happens to the motion of a wave when it reaches a boundary?

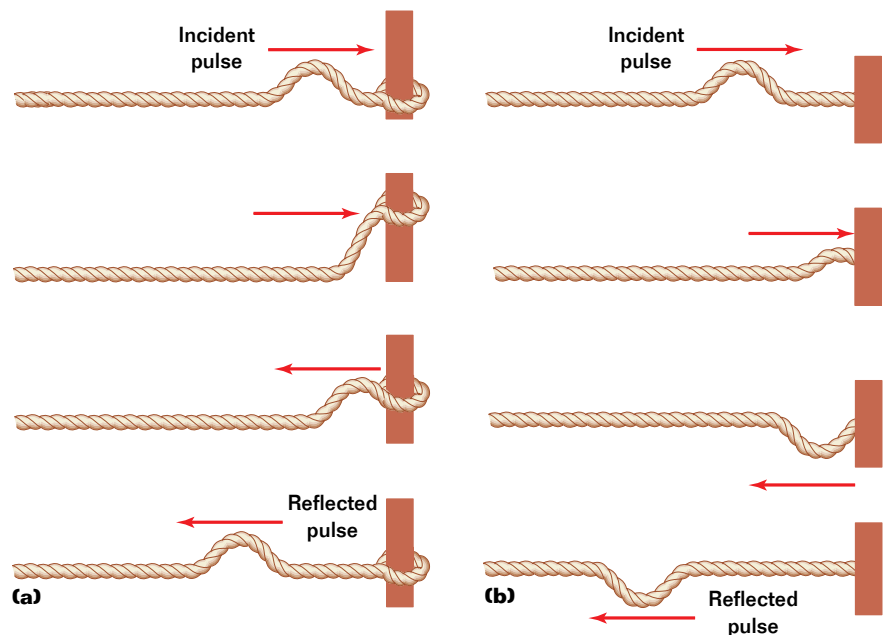
### At a free boundary, waves are reflected

Consider a pulse wave traveling on a stretched rope whose end forms a ring around a post, as shown in **Figure 19(a)**. We will assume that the ring is free to slide along the post without friction.

As the pulse travels to the right, each point of the rope moves up once and then back down. When the pulse reaches the boundary, the rope is free to move up as usual, and it pulls the ring up with it. Then, the ring is pulled back down by the tension in the rope. The movement of the rope at the post is similar to the movement that would result if someone were to whip the rope upward to send a pulse to the left, which would cause a pulse to travel back along the rope to the left. This is called *reflection*. Note that the reflected pulse is upright and has the same amplitude as the incident pulse.

### At a fixed boundary, waves are reflected and inverted

Now consider a pulse traveling on a stretched rope that is fixed at one end, as in **Figure 19(b)**. When the pulse reaches the wall, the rope exerts an upward force on the wall, and the wall in turn exerts an equal and opposite reaction force on the rope. This downward force on the rope causes a displacement in the direction opposite the displacement of the original pulse. As a result, the pulse is inverted after reflection.



**Figure 19**

(a) When a pulse travels down a rope whose end is free to slide up the post, the pulse is reflected from the free end. (b) When a pulse travels down a rope that is fixed at one end, the reflected pulse is inverted.

## STANDING WAVES

Consider a string that is attached on one end to a rigid support and that is shaken up and down in a regular motion at the other end. The regular motion produces waves of a certain frequency, wavelength, and amplitude traveling down the string. When the waves reach the other end, they are reflected back toward the oncoming waves. If the string is vibrated at exactly the right frequency, a **standing wave**—a resultant wave pattern that appears to be stationary on the string—is produced. The standing wave consists of alternating regions of constructive and destructive interference.

### Standing waves have nodes and antinodes

**Figure 20(a)** shows four possible standing waves for a given string length. The points at which complete destructive interference happens are called **nodes**. There is no motion in the string at the nodes. But midway between two adjacent nodes, the string vibrates with the largest amplitude. These points are called **antinodes**.

**Figure 20(b)** shows the oscillation of the second case shown in **Figure 20(a)** during half a cycle. All points on the string oscillate vertically with the same frequency, except for the nodes, which are stationary. In this case, there are three nodes (*N*) and two antinodes (*A*), as illustrated in the figure.

### standing wave

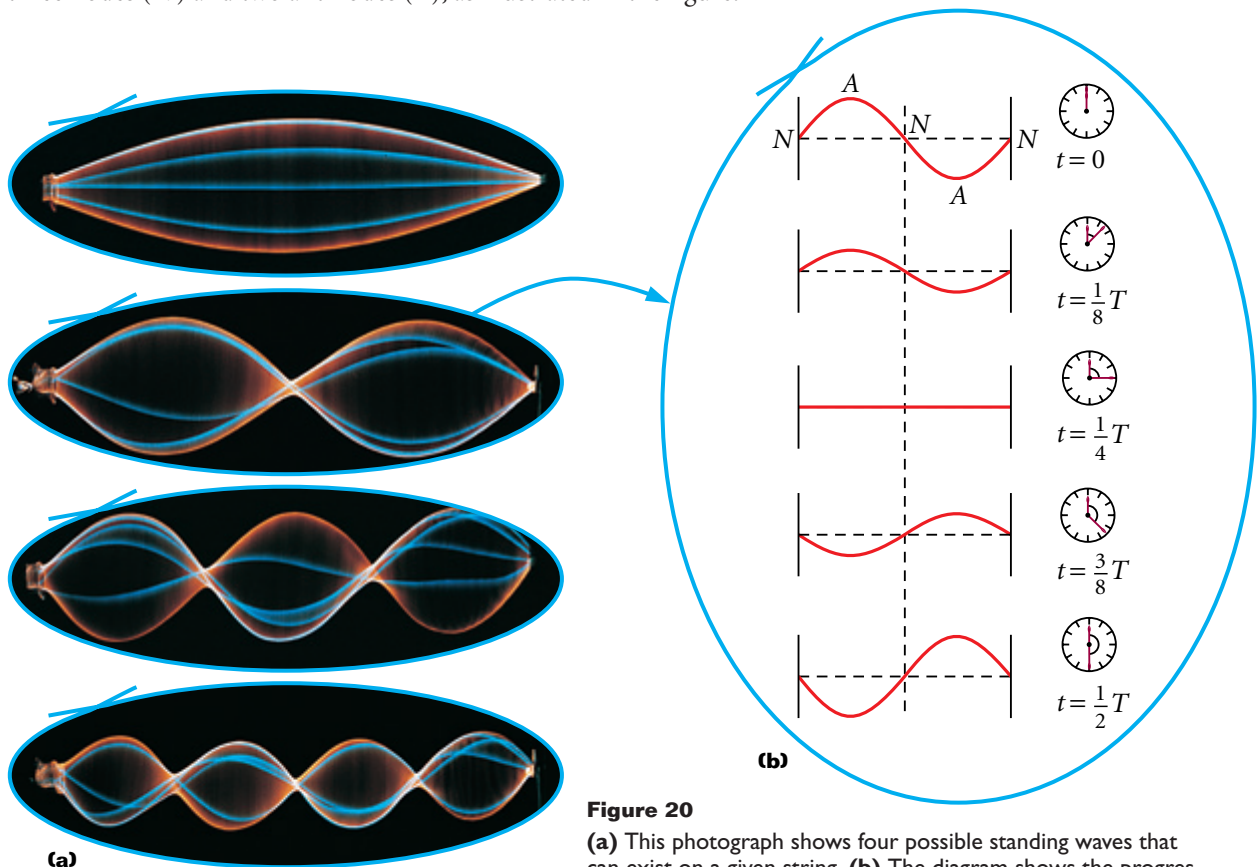
a wave pattern that results when two waves of the same frequency, wavelength, and amplitude travel in opposite directions and interfere

### node

a point in a standing wave that maintains zero displacement

### antinode

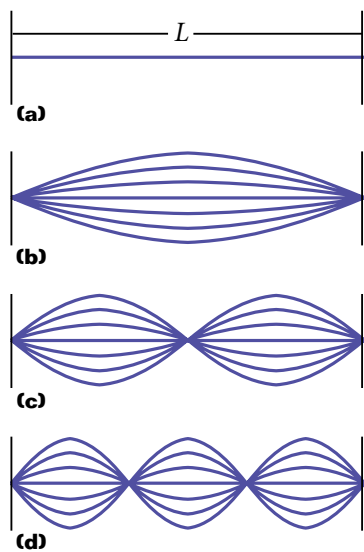
a point in a standing wave, halfway between two nodes, at which the largest displacement occurs



**Figure 20**

(a) This photograph shows four possible standing waves that can exist on a given string. (b) The diagram shows the progression of the second standing wave for one-half of a cycle.





**Figure 21**

Only certain frequencies produce standing waves on this fixed string. The wavelength of these standing waves depends on the string length. Possible wavelengths include  $2L$  (b),  $L$  (c), and  $\frac{2}{3}L$  (d).

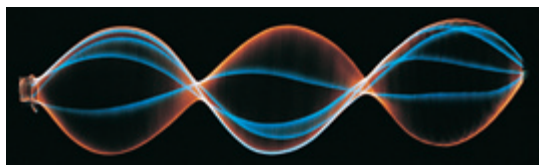
Only certain frequencies, and therefore wavelengths, produce standing wave patterns. **Figure 21** shows standing waves for a given string length. In each case, the curves represent the position of the string at different instants of time. If the string were vibrating rapidly, the several positions would blur together and give the appearance of loops, like those shown in the diagram. A single loop corresponds to either a crest or trough alone, while two loops correspond to a crest and a trough together, or one wavelength.

The ends of the string must be nodes because these points cannot vibrate. As you can see in **Figure 21**, a standing wave can be produced for any wavelength that allows both ends of the string to be nodes. For example, in **Figure 21(b)**, each end is a node, and there are no nodes in between. Because a single loop corresponds to either a crest or trough alone, this standing wave corresponds to one-half of a wavelength. Thus, the wavelength in this case is equal to twice the string length ( $2L$ ).

The next possible standing wave, shown in **Figure 21(c)**, has three nodes: one at either end and one in the middle. In this case, there are two loops, which correspond to a crest and a trough. Thus, this standing wave has a wavelength equal to the string length ( $L$ ). The next case, shown in **Figure 21(d)**, has a wavelength equal to two-thirds of the string length ( $\frac{2}{3}L$ ), and the pattern continues. Wavelengths between the values shown here do not produce standing waves because they allow only one end of the string to be a node.

## SECTION REVIEW

1. A wave of amplitude 0.30 m interferes with a second wave of amplitude 0.20 m. What is the largest resultant displacement that may occur?
2. A string is rigidly attached to a post at one end. Several pulses of amplitude 0.15 m sent down the string are reflected at the post and travel back down the string without a loss of amplitude. What is the amplitude at a point on the string where the maximum displacement points of two pulses cross? What type of interference is this?
3. How would your answer to item 2 change if the same pulses were sent down a string whose end is free? What type of interference is this?
4. A stretched string fixed at both ends is 2.0 m long. What are three wavelengths that will produce standing waves on this string? Name at least one wavelength that would not produce a standing wave pattern, and explain your answer.
5. **Interpreting Graphics** Look at the standing wave shown in **Figure 22**. How many nodes does this wave have? How many antinodes?



**Figure 22**

**KEY IDEAS****Section 1 Simple Harmonic Motion**

- In simple harmonic motion, restoring force is proportional to displacement.
- A mass-spring system vibrates with simple harmonic motion, and the spring force is given by Hooke's law.
- For small angles of displacement ( $<15^\circ$ ), a simple pendulum swings with simple harmonic motion.
- In simple harmonic motion, restoring force and acceleration are maximum at maximum displacement and speed is maximum at equilibrium.

**Section 2 Measuring Simple Harmonic Motion**

- The period of a mass-spring system depends only on the mass and the spring constant. The period of a simple pendulum depends only on the string length and the free-fall acceleration.
- Frequency is the inverse of period.

**Section 3 Properties of Waves**

- As a wave travels, the particles of the medium vibrate around an equilibrium position.
- In a transverse wave, vibrations are *perpendicular* to the direction of wave motion. In a longitudinal wave, vibrations are *parallel* to the direction of wave motion.
- Wave speed equals frequency times wavelength.

**Section 4 Wave Interactions**

- If two or more waves are moving through a medium, the resultant wave is found by adding the individual displacements together point by point.
- Standing waves are formed when two waves that have the same frequency, amplitude, and wavelength travel in opposite directions and interfere.

**KEY TERMS**

**simple harmonic motion**  
(p. 369)

**amplitude** (p. 376)

**period** (p. 376)

**frequency** (p. 376)

**medium** (p. 382)

**mechanical wave** (p. 382)

**transverse wave** (p. 384)

**crest** (p. 384)

**trough** (p. 384)

**wavelength** (p. 384)

**longitudinal wave** (p. 385)

**constructive interference**  
(p. 390)

**destructive interference**  
(p. 391)

**standing wave** (p. 393)

**node** (p. 393)

**antinode** (p. 393)

**Variable Symbols**

Quantities		Units	
$F_{\text{elastic}}$	spring force	N	newtons
$k$	spring constant	N/m	newtons/meter
$T$	period	s	seconds
$f$	frequency	Hz	hertz = $s^{-1}$
$\lambda$	wavelength	m	meters

**PROBLEM SOLVING**

See **Appendix D: Equations** for a summary of the equations introduced in this chapter. If you need more problem-solving practice, see **Appendix I: Additional Problems**.

**SIMPLE HARMONIC MOTION****Review Questions**

1. What characterizes an object's motion as simple harmonic?
2. List four examples of simple harmonic motion.
3. Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain.
4. A pendulum is released  $40^\circ$  from its resting position. Is its motion simple harmonic?
5. April is about to release the bob of a pendulum. Before she lets go, what sort of potential energy does the bob have? How does the energy of the bob change as it swings through one full cycle of motion?

**Conceptual Questions**

6. An ideal mass-spring system vibrating with simple harmonic motion would oscillate indefinitely. Explain why.
7. In a simple pendulum, the weight of the bob can be divided into two components, one tangent to the bob's direction of motion and the other perpendicular to the bob's direction of motion. Which of these is the restoring force, and why?

**Practice Problems**

For problems 8–9, see Sample Problem A.

8. Janet wants to find the spring constant of a given spring, so she hangs the spring vertically and attaches a 0.40 kg mass to the spring's other end. If the spring stretches 3.0 cm from its equilibrium position, what is the spring constant?
9. In preparing to shoot an arrow, an archer pulls a bowstring back 0.40 m by exerting a force that increases uniformly from 0 to 230 N. What is the equivalent spring constant of the bow?

**PERIOD AND FREQUENCY****Review Questions**

10. A child swings on a playground swing. How many times does the child swing through the swing's equilibrium position during the course of a single period of motion?
11. What is the total distance traveled by an object moving back and forth in simple harmonic motion in a time interval equal to its period when its amplitude is equal to  $A$ ?
12. How is the period of a simple harmonic vibration related to its frequency?

**Conceptual Questions**

13. What happens to the period of a simple pendulum when the pendulum's length is doubled? What happens when the suspended mass is doubled?
14. A pendulum bob is made with a ball filled with water. What would happen to the frequency of vibration of this pendulum if a hole in the ball allowed water to slowly leak out? (Treat the pendulum as a simple pendulum.)
15. If a pendulum clock keeps perfect time at the base of a mountain, will it also keep perfect time when moved to the top of the mountain? Explain.
16. If a grandfather clock is running slow, how can you adjust the length of the pendulum to correct the time?
17. A simple pendulum can be used as an altimeter on a plane. How will the period of the pendulum vary as the plane rises from the ground to its final cruising altitude?
18. Will the period of a vibrating mass-spring system on Earth be different from the period of an identical mass-spring system on the moon? Why or why not?

## Practice Problems

For problems 19–20, see Sample Problem B.

19. Find the length of a pendulum that oscillates with a frequency of 0.16 Hz.
20. A pendulum that moves through its equilibrium position once every 1.000 s is sometimes called a *seconds pendulum*.
  - a. What is the period of any seconds pendulum?
  - b. In Cambridge, England, a seconds pendulum is 0.9942 m long. What is the free-fall acceleration in Cambridge?
  - c. In Tokyo, Japan, a seconds pendulum is 0.9927 m long. What is the free-fall acceleration in Tokyo?

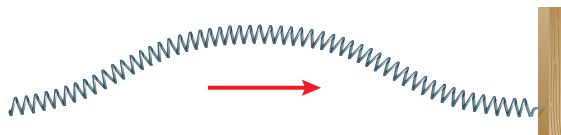
For problem 21, see Sample Problem C.

21. A spring with a spring constant of  $1.8 \times 10^2$  N/m is attached to a 1.5 kg mass and then set in motion.
  - a. What is the period of the mass-spring system?
  - b. What is the frequency of the vibration?

## PROPERTIES OF WAVES

### Review Questions

22. What is common to all waves?
23. How do transverse and longitudinal waves differ?
24. The figure below depicts a pulse wave traveling on a spring.
  - a. In which direction are the particles of the medium vibrating?
  - b. Is this wave transverse or longitudinal?

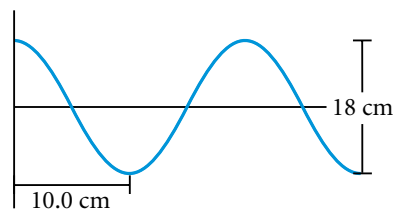


25. In a stretched spring, several coils are pinched together and others are spread farther apart than usual. What sort of wave is this?
26. How far does a wave travel in one period?

27. If you shook the end of a rope up and down three times each second, what would be the period of the waves set up in the rope? What would be the frequency?
28. Give three examples of mechanical waves. How are these different from electromagnetic waves, such as light waves?

### Conceptual Questions

29. How does a single point on a string move as a transverse wave passes by that point?
30. What happens to the wavelength of a wave on a string when the frequency is doubled? What happens to the speed of the wave?
31. Why do sound waves need a medium through which to travel?
32. Two tuning forks with frequencies of 256 Hz and 512 Hz are struck. Which of the sounds will move faster through the air?
33. What is one advantage of transferring energy by electromagnetic waves?



34. A wave traveling in the positive  $x$  direction with a frequency of 25.0 Hz is shown in the figure above. Find the following values for this wave:
  - a. amplitude
  - b. wavelength
  - c. period
  - d. speed

### Practice Problems

For problem 35, see Sample Problem D.

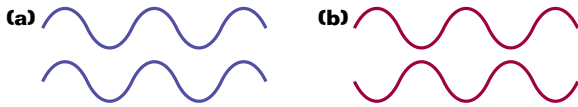
35. Microwaves travel at the speed of light,  $3.00 \times 10^8$  m/s. When the frequency of microwaves is  $9.00 \times 10^9$  Hz, what is their wavelength?



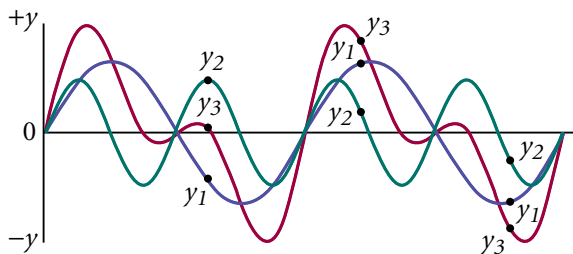
## WAVE INTERACTIONS

### Review Questions

36. Using the superposition principle, draw the resultant waves for each of the examples below.



37. What is the difference between constructive interference and destructive interference?
38. Which one of the waveforms shown below is the resultant waveform?



39. Anthony sends a series of pulses of amplitude 24 cm down a string that is attached to a post at one end. Assuming the pulses are reflected with no loss of amplitude, what is the amplitude at a point on the string where two pulses are crossing if
- the string is rigidly attached to the post?
  - the end at which reflection occurs is free to slide up and down?

### Conceptual Questions

40. Can more than two waves interfere in a given medium?
41. What is the resultant displacement at a position where destructive interference is complete?
42. When two waves interfere, can the resultant wave be larger than either of the two original waves? If so, under what conditions?
43. Which of the following wavelengths will produce standing waves on a string that is 3.5 m long?
- 1.75 m
  - 3.5 m
  - 5.0 m
  - 7.0 m

## MIXED REVIEW

44. In an arcade game, a 0.12 kg disk is shot across a frictionless horizontal surface by being compressed against a spring and then released. If the spring has a spring constant of 230 N/m and is compressed from its equilibrium position by 6.0 cm, what is the magnitude of the spring force on the disk at the moment it is released?

45. A child's toy consists of a piece of plastic attached to a spring, as shown at right. The spring is compressed against the floor a distance of 2.0 cm and released. If the spring constant is 85 N/m, what is the magnitude of the spring force acting on the toy at the moment it is released?



46. You dip your finger into a pan of water twice each second, producing waves with crests that are separated by 0.15 m. Determine the frequency, period, and speed of these water waves.
47. A sound wave traveling at 343 m/s is emitted by the foghorn of a tugboat. An echo is heard 2.60 s later. How far away is the reflecting object?
48. The notes produced by a violin range in frequency from approximately 196 Hz to 2637 Hz. Find the possible range of wavelengths in air produced by this instrument when the speed of sound in air is 340 m/s.
49. What is the free-fall acceleration in a location where the period of a 0.850 m long pendulum is 1.86 s?
50. Yellow light travels through a certain glass block at a speed of  $1.97 \times 10^8$  m/s. The wavelength of the light in this particular type of glass is  $3.81 \times 10^{-7}$  m (381 nm). What is the frequency of the yellow light?
51. A certain pendulum clock that works perfectly on Earth is taken to the moon, where  $a_g = 1.63 \text{ m/s}^2$ . If the clock is started at 12:00 A.M., what will it read after 24.0 h have passed on Earth?

## Alternative Assessment

1. Design an experiment to compare the spring constant and period of oscillation of a system built with two (or more) springs connected in two ways: in series (attached end to end) and in parallel (one end of each spring anchored to a common point). If your teacher approves your plan, obtain the necessary equipment and perform the experiment.
2. The rule that the period of a pendulum is determined by its length is a good approximation for amplitudes below  $15^\circ$ . Design an experiment to investigate how amplitudes of oscillation greater than  $15^\circ$  affect the motion of a pendulum.  
List what equipment you would need, what measurements you would perform, what data you would record, and what you would calculate. If your teacher approves your plan, obtain the necessary equipment and perform the experiment.
3. Research earthquakes and different kinds of seismic waves. Create a presentation about earthquakes that includes answers to the following questions as well as additional information: Do earthquakes travel through oceans? What is transferred from place to place as seismic waves propagate? What determines their speed?
4. Identify examples of periodic motion in nature. Create a chart describing the objects involved, their path of motion, their periods, and the forces involved. Which of the periodic motions are harmonic and which are not?

## Graphing Calculator Practice



### Pendulum

Would a pendulum have the same period of oscillation on Mars, Venus, or Neptune? A pendulum's period, as you learned earlier in this chapter, is described by the following equation:

$$T = 2\pi \sqrt{\frac{L}{a_g}}$$

In this equation,  $T$  is the period,  $L$  is the length of the pendulum, and  $a_g$  is the free-fall acceleration ( $9.81 \text{ m/s}^2$  on Earth's surface). This equation can be rearranged to solve for  $L$  if  $T$  is known.

$$L = \frac{a_g T^2}{4\pi^2}$$

In this graphing calculator activity, you will enter the period of a pendulum on Earth's surface. The calculator will use the previous equation to determine  $L$ , the length of the pendulum. The calculator will then use this length to display a graph showing how the period of this pendulum changes as free-fall acceleration changes. You will use this graph to find the period of a pendulum on various planets.

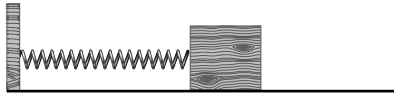
Visit [go.hrw.com](http://go.hrw.com) and type in the keyword **HF6VIBX** to find this graphing calculator activity. Refer to **Appendix B** for instructions on downloading the program for this activity.



# Standardized Test Prep

## MULTIPLE CHOICE

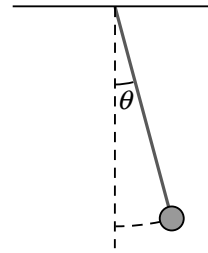
Base your answers to questions 1–6 on the information below.



A mass is attached to a spring and moves with simple harmonic motion on a frictionless horizontal surface, as shown above.

- In what direction does the restoring force act?
  - to the left
  - to the right
  - to the left or to the right depending on whether the spring is stretched or compressed
  - perpendicular to the motion of the mass
- If the mass is displaced  $-0.35$  m from its equilibrium position, the restoring force is  $7.0$  N. What is the spring constant?
  - $-5.0 \times 10^{-2}$  N/m
  - $-2.0 \times 10^1$  N/m
  - $5.0 \times 10^{-2}$  N/m
  - $2.0 \times 10^1$  N/m
- In what form is the energy in the system when the mass passes through the equilibrium position?
  - elastic potential energy
  - gravitational potential energy
  - kinetic energy
  - a combination of two or more of the above
- In what form is the energy in the system when the mass is at maximum displacement?
  - elastic potential energy
  - gravitational potential energy
  - kinetic energy
  - a combination of two or more of the above
- Which of the following does *not* affect the period of the mass-spring system?
  - mass
  - spring constant
  - amplitude of vibration
  - All of the above affect the period.
- If the mass is  $48$  kg and the spring constant is  $12$  N/m, what is the period of the oscillation?
  - $8\pi$  s
  - $4\pi$  s
  - $\pi$  s
  - $\frac{\pi}{2}$  s

Base your answers to questions 7–10 on the information below.

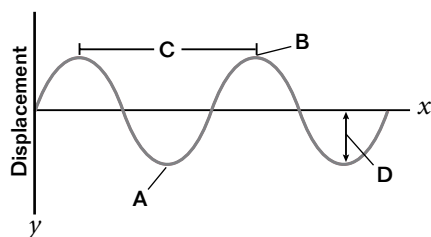


A pendulum bob hangs from a string and moves with simple harmonic motion, as shown above.

- What is the restoring force in the pendulum?
  - the total weight of the bob
  - the component of the bob's weight tangent to the motion of the bob
  - the component of the bob's weight perpendicular to the motion of the bob
  - the elastic force of the stretched string
- Which of the following does *not* affect the period of the pendulum?
  - the length of the string
  - the mass of the pendulum bob
  - the free-fall acceleration at the pendulum's location
  - All of the above affect the period.

9. If the pendulum completes exactly 12 cycles in 2.0 min, what is the frequency of the pendulum?
- A. 0.10 Hz                      C. 6.0 Hz  
B. 0.17 Hz                      D. 10 Hz
10. If the pendulum's length is 2.00 m and  $a_g = 9.80 \text{ m/s}^2$ , how many complete oscillations does the pendulum make in 5.00 min?
- F. 1.76                          H. 106  
G. 21.6                         J. 239

Base your answers to questions 11–13 on the graph below.



11. What kind of wave does this graph represent?
- A. transverse wave  
B. longitudinal wave  
C. electromagnetic wave  
D. pulse wave
12. Which letter on the graph is used for the wavelength?
- F. A                                H. C  
G. B                                J. D
13. Which letter on the graph is used for a trough?
- A. A                                C. C  
B. B                                D. D

Base your answers to questions 14–15 on the passage below.

A wave with an amplitude of 0.75 m has the same wavelength as a second wave with an amplitude of 0.53 m. The two waves interfere.

14. What is the amplitude of the resultant wave if the interference is constructive?
- F. 0.22 m                      H. 0.75 m  
G. 0.53 m                      J. 1.28 m

15. What is the amplitude of the resultant wave if the interference is destructive?
- A. 0.22 m                      C. 0.75 m  
B. 0.53 m                      D. 1.28 m
16. Two successive crests of a transverse wave are 1.20 m apart. Eight crests pass a given point every 12.0 s. What is the wave speed?
- F. 0.667 m/s                  H. 1.80 m/s  
G. 0.800 m/s                  J. 9.60 m/s

## SHORT RESPONSE

17. Green light has a wavelength of  $5.20 \times 10^{-7} \text{ m}$  and a speed in air of  $3.00 \times 10^8 \text{ m/s}$ . Calculate the frequency and the period of the light.
18. What kind of wave does not need a medium through which to travel?
19. List three wavelengths that could form standing waves on a 2.0 m string that is fixed at both ends.

## EXTENDED RESPONSE

20. A visitor to a lighthouse wishes to find out the height of the tower. The visitor ties a spool of thread to a small rock to make a simple pendulum. Then, the visitor hangs the pendulum down a spiral staircase in the center of the tower. The period of oscillation is 9.49 s. What is the height of the tower? Show all of your work.
21. A harmonic wave is traveling along a rope. The oscillator that generates the wave completes 40.0 vibrations in 30.0 s. A given crest of the wave travels 425 cm along the rope in a period of 10.0 s. What is the wavelength? Show all of your work.

**Test TIP** Take a little time to look over a test before you start. Look for questions that may be easy for you to answer, and answer those first. Then, move on to the harder questions.

# Simple Harmonic Motion of a Pendulum

## Design Your Own

### OBJECTIVES

- **Construct** simple pendulums, and find their periods.
- **Calculate** the value for  $a_g$ , the free-fall acceleration.
- **Examine** the relationships between length, mass, and period for different pendulums.

### MATERIALS LIST

- balance
- cord
- meterstick
- pendulum bobs
- pendulum clamp
- protractor
- stopwatch
- support stand

The period of a pendulum is the time required for the pendulum to complete one cycle. In this lab, you will construct models of a simple pendulum using different masses and lengths of cord. You will design an experiment to measure the period of each model and to determine how the period depends on length and mass. Your experiment should include several trials at a constant mass but at different lengths, and several more trials at constant length but with different masses. In the Analysis, you will also use the period and the length of the cord for each trial to calculate the free-fall acceleration,  $a_g$ , at your geographical location.

### SAFETY



- **Tie back long hair, secure loose clothing, and remove loose jewelry to prevent their getting caught in moving parts or pulleys. Put on goggles.**
- **Attach masses to the thread and the thread to clamps securely. Swing masses in areas free of people and obstacles. Swinging or dropped masses can cause serious injury.**

### PROCEDURE

1. Study the materials provided, and design an experiment to meet the goals stated above.
2. Write out your lab procedure, including a detailed description of the measurements to take during each step and the number of trials to perform. You may use **Figure 1** as a guide to a possible setup. You should keep the amplitude of the swing less than  $15^\circ$  in each trial.
3. Ask your teacher to approve your procedure.
4. Follow all steps of your procedure.
5. Clean up your work area. Put equipment away safely so that it is ready to be used again.



## ANALYSIS

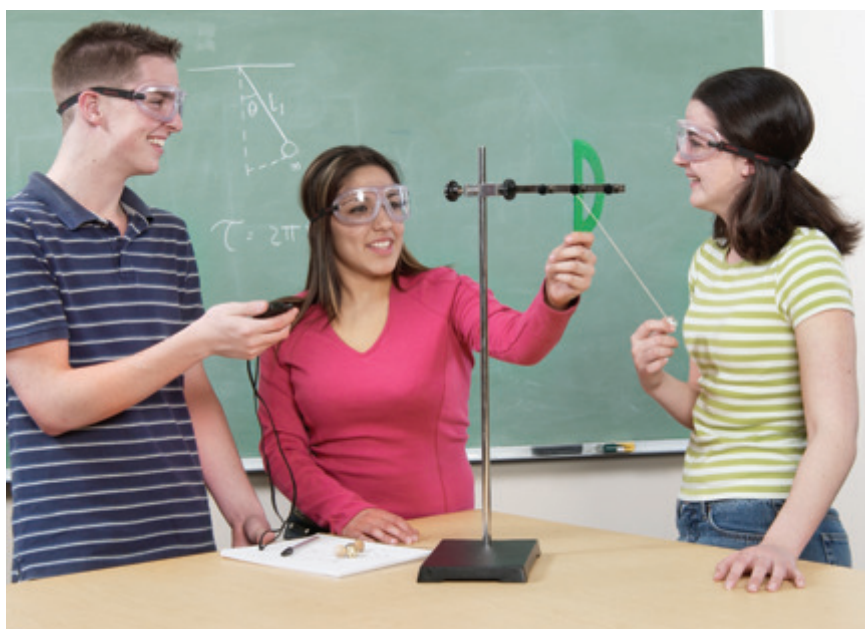
- 1. Organizing Data** For each trial, calculate the period of the pendulum.
- 2. Organizing Data** Calculate the value for the free-fall acceleration,  $a_g$ , for each trial. Use the equation for the period of a simple pendulum, rearranged to solve for  $a_g$ .
- 3. Constructing Graphs** Plot the following graphs:
  - a. the period vs. the length (for constant-mass trials)
  - b. the period vs. the mass of the bob (for constant-length trials)
  - c. the period vs. the square root of the length (for constant-mass trials)

## CONCLUSIONS

- 4. Evaluating Results** Use  $9.81 \text{ m/s}^2$  as the accepted value for  $a_g$ .
  - a. Compute the absolute error for each trial using the following equation:
$$\text{absolute error} = |\text{experimental} - \text{accepted}|$$
  - b. Compute the relative error for each trial using the following equation:

$$\text{relative error} = \frac{(\text{experimental} - \text{accepted})}{\text{accepted}}$$

- 5. Drawing Conclusions** Based on your data and your graphs, how does the mass of the pendulum bob affect the period of vibration?
- 6. Drawing Conclusions** Based on your data and your graphs, how does the length of the pendulum affect the period of vibration?



**Figure 1**

- Hold the bob so that the cord is perfectly straight while you measure the angle.
- Release the bob gently so that it swings smoothly. Practice counting and timing cycles to get good results.

1780

# Physics and Its World *Timeline 1785–1830*



**1789** – The storming of the Bastille marks the climax of the French Revolution.



**1798**  $Q = mc_p \Delta T$

**Benjamin Thompson** (Count Rumford) demonstrates that energy transferred as heat results from mechanical processes, rather than the release of caloric, the heat fluid that has been widely believed to exist in all substances.

1790



**1796** – Edward Jenner develops the smallpox vaccine.

1800



**1800**

$$\Delta V = \frac{\Delta PE_{electric}}{q}$$

**Alessandro Volta** develops the first current-electricity cell using alternating plates of silver and zinc.

1810

**1801**

$$m\lambda = d(\sin \theta)$$

**Thomas Young** demonstrates that light rays interfere, providing the first substantial support for a wave theory of light.



**1804** – Saint-Domingue, under the control of the French-African majority led by **Toussaint-Louverture**, becomes the independent Republic of Haiti. Over the next two decades most of Europe's western colonies become independent.



**1804** – **Richard Trevithick** builds and tests the first steam locomotive. It pulls 10 tons along a distance of 15 km at a speed of 8 km/h.





**1811** – Mathematician **Sophie Germain** writes the first of three papers on the mathematics of vibrating surfaces. She later addresses one of the most famous problems in mathematics—Fermat’s last theorem—proving it to be true for a wide range of conditions.

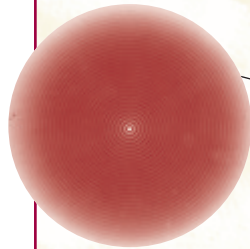


**1810** – **Kamehameha I** unites the Hawaiian islands under a monarchy.

**1814**

$$\sin \theta = \frac{m\lambda}{a}$$

**Augustin Fresnel** begins his research in optics, the results of which will confirm and explain **Thomas Young’s** discovery of interference and will firmly establish the wave model of light first suggested by **Christiaan Huygens** over a century earlier.



**1820**

$$F_{\text{magnetic}} = BI\ell$$

**Hans Christian Oersted** demonstrates that an electric current produces a magnetic field. (**Gian Domenico Romagnosi**, an amateur scientist, discovered the effect 18 years earlier, but at the time attracted no attention.) **André-Marie Ampère** repeats Oersted’s experiment and formulates the law of electromagnetism that today bears his name.

**1818** – **Mary Shelley** writes *Frankenstein, or the Modern Prometheus*. Primarily thought of as a horror novel, the book’s emphasis on science and its moral consequences also qualifies it as the first “science fiction” novel.



**1826** – **Katsushika Hokusai** begins his series of prints *Thirty-Six Views of Mount Fuji*.



**1830** – **Hector Berlioz** composes his *Symphonie Fantastique*, one of the first Romantic works for large orchestra that tells a story with music.

1800

1810

1820

1830